LOW NOISE OSCILLATOR DESIGN

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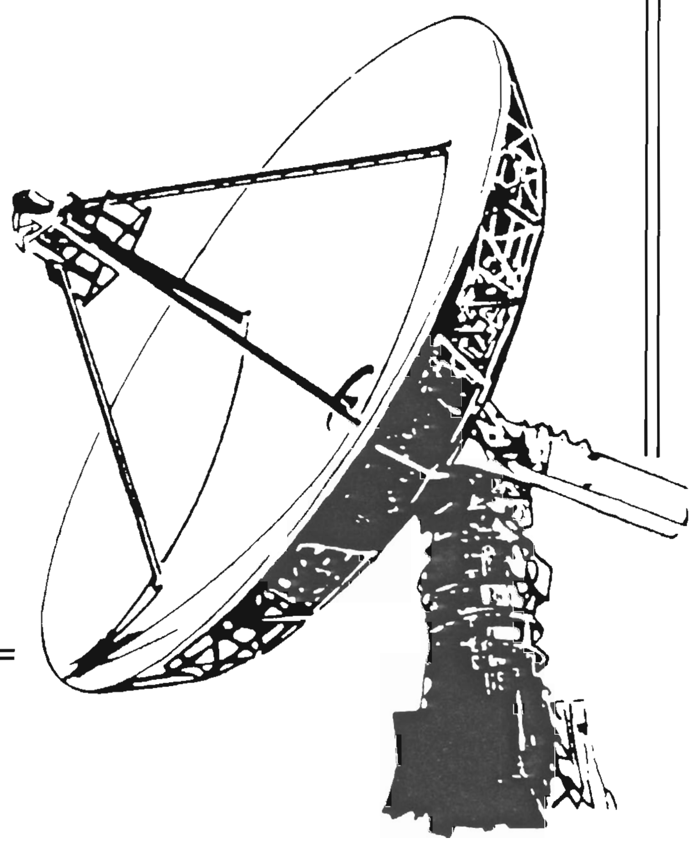
RF & Microwave Measurement Symposium and Exhibition



LOW NOISE OSCILLATOR DESIGN

I. Spectral Purity

A. What is spectral purity in oscillators?

B. What determines spectral purity?

II. Low Noise Oscillator Design

A. Establish objectives

B. Select a resonator

1. Measure: Qu, 1/f noise, AM-FM conversion
2. Analyze phase noise

C. Select a circuit topology

D. Select an active device

E. Select matching networks

F. Measure:

1. Open loop gain/phase
2. Closed loop gain/phase

III. Oscillator Computer Analysis

A. Open loop

B. Closed loop

IV. Other Noise Mechanisms

A. Spurious modes

B. Upconverted noise

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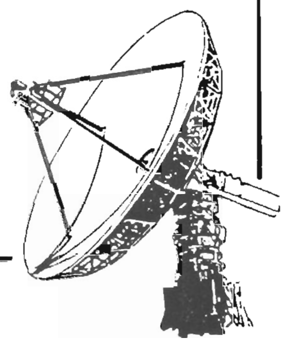

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LOW NOISE OSCILLATOR DESIGN

SPECTRAL PURITY
- A. What is spectral purity in oscillators?
- B. What determines spectral purity?

LOW NOISE OSCILLATOR DESIGN
- A. Establish objectives
- B. Select a resonator
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OSCILLATOR COMPUTER ANALYSIS
- A. Open loop
- B. Closed loop

OTHER NOISE MECHANISMS
- A. Spurious modes
- B. Upconverted noise

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SPECTRAL PURITY DEFINITIONS

$$V(t) = \cos w_o t$$

$$V(w) = \delta(w_o) + \delta(w_o)$$

$$V(t) = \underbrace{[1+e_A(t)]}_{\text{AM Noise}} \cos [w_o t + \underbrace{\phi(t)]}_{\text{PM Noise}}$$

$$\mathcal{L}(f_m) = \frac{\text{SSB Noise Power in a 1 Hz Bw } f_m \text{ Hz from Carrier}}{\text{Total Signal Power}}$$

$$\mathcal{L}(f_m) = \frac{S\phi(f_m)}{4}$$

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Spectral Purity describes the degree of degradation from a perfect impulse in the frequency domain:

$$V(t) = \cos \omega_0 t$$

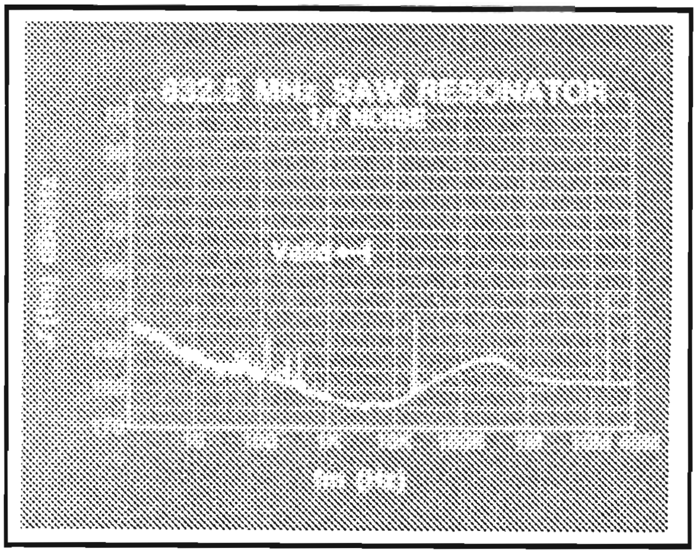
$$V(\omega) = \delta(\omega_0) + \delta(-\omega_0)$$

Real signals have some noise associated

$$V(t) = [1 + e_A(t)] \cos [\omega_0 t + \phi(t)]$$

We will focus primarily on phase noise (PM) components. $\mathcal{L}(f_m)$ describes the ratio of SSB power in a 1 Hz B.W. due to phase noise, offset f_m Hz, from the carrier, to the total signal power.

Ref. 2, 14, 15



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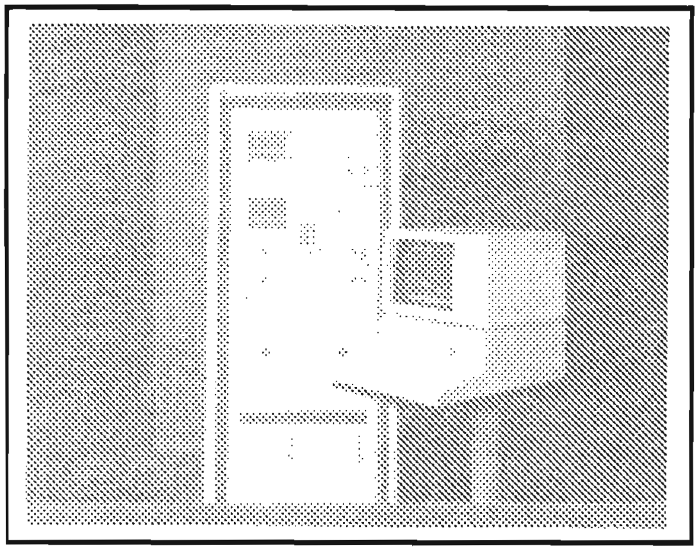
This is a typical phase noise measurement. This oscillator uses a 832.5 MHz surface acoustic wave resonator as the resonator. How good is this performance? Could it be better? What are the limits to the noise performance of this oscillator? What are the significant contributors to its noise?



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The previous phase noise plot was measured on Hewlett-Packard's 3047 phase noise measurement system. This system has the capability of measuring noise as low as ~−170 dBc. It covers the frequency range 10 MHz to 18 GHz.

SPECTRAL PURITY

KEY PARAMETERS

- NOISE : NF, thermal, 1/f, AM-FM, bias upconversion, unflat gain, unflat gain compression
- SIGNAL LEVEL: P_{AVS}
- LOADED Q_L

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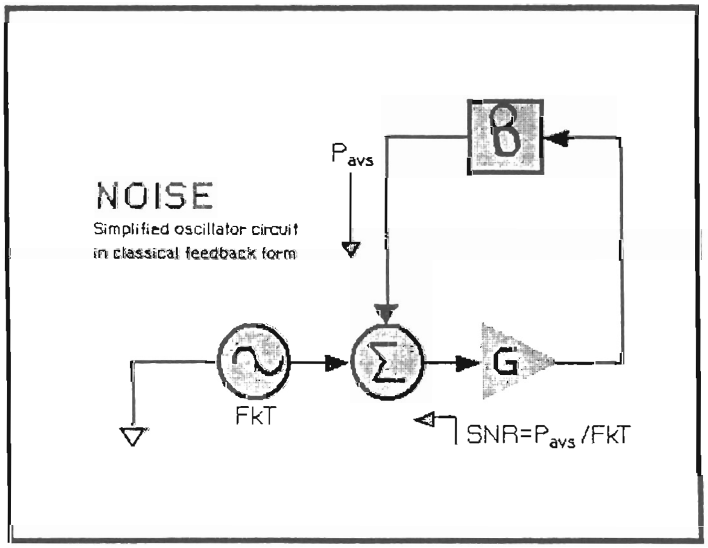
Key parameters that will be important in our discussion of phase noise in oscillators include noise iteself as evidenced by the noise figure of the active devices and circuits used in the oscillator, 1/f noise of active devices and resonators, AM-FM conversion of noise, upconversion of bias noise, and unflat gain.

Signal levels in the circuit are important. Higher signal levels lead to higher signal to noise ratios and thus better phase noise.

We will see that loaded resonator Q will determine phase noise close to the carrier and that increasing loaded Q will improve phase noise close to the carrier.

We can model an oscillator in the classical feedback form with an amplifier with gain G and feedback β which includes the resonator. For os-oscillation at $f = f_0$, two conditions must be satisfied:



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1. Loop gain is greater than one at f_0.

$$|G\beta| > 1 \text{ at } f = f_0$$

2. Phase shift around the loop = 0

$$\angle G\beta = 0 \text{ at } f = f_0$$

In the interest of preventing spurious oscillations at undesired frequencies, two other conditions should be met:

$$|G\beta| < 1 \text{ at } f \neq f_0$$

and

$$\Gamma_{node} < 1 \text{ for all nodes at } f \neq f_0$$

where Γ is the reflection coefficient looking into any node. (Meeting this condition at the collector and base nodes is usually sufficient.)

Signal to noise ratio at the input to the amplifier is P_{avs}/FkT where P_{avs} is the power available at the input of the amplifier and F is the noise figure of the amplifier.

SPECTRAL PURITY

KEY RELATIONSHIPS

$$\mathscr{L}(f_m) = -10 \log \frac{FkT}{P_{avs}}\left[1 + \left(\frac{1}{f_m}\frac{f_0}{2Q_L}\right)^2\right] \frac{1}{2}$$

$$\mathscr{L}(f_m) = -SNR_i \; -3dB \; \underbrace{+10 \log\left[1 + \left(\frac{1}{f_m}\frac{f_0}{2Q_L}\right)^2\right]}_{\text{CLOSED LOOP PEAKING}}$$

$$\mathscr{L}(f_m) = -P_{AVS}\,(dBm) + NF(dB) - 177 dBc/Hz + PEAKING(dB)$$

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We assume that the signal to noise ratio at the input P_{avs}/FkT causes both amplitude and phase noise in equal amounts. For frequencies far from resonance, f_0, where loop gain ≪1, phase noise relative to the carrier will be

$$f_m = \frac{1}{2}\frac{FkT}{P_{avs}}$$

Thus

$$\mathscr{L}(f_m) = -SNR_i - 3 \text{ dB}$$

where

$$SNR_i = 10 \log \left(\frac{P_{avs}}{FkT}\right)$$

for $f_m \gg$ loop bandwidth.

Close to the carrier, loop gain peaking will cause amplification of this noise. Let's first understand loaded resonator Q:

$$Q_L = \frac{f_0}{2}\left.\frac{\partial\angle(G\beta)}{\partial f}\right|_{f=f_0}$$

where

$$\frac{\partial\angle(G\beta)}{\partial f} = \text{loop gain phase slope.}$$

Loaded Q determines the open loop bandwidth of the feedback loop used to represent the oscillator. Inside the bandwidth,

$$\frac{f_0}{2Q_L},$$

when the loop is closed loop peaking increases phase noise. A first order approximation of phase noise is then

$$L(f_m) = \frac{1}{2}\frac{FkT}{P_{avs}}\left[1 + \left(\frac{1}{f_m}\frac{f_0}{2Q_L}\right)^2\right]$$

where

$L(f_m)$ = the ratio of SSB noise power due to PM in a 1 Hz bandwidth (centered f_m Hz off the carrier) to total signal power;

F = the noise factor of the active device;

k = Boltzmann's constant; = 1.38×10^{-23} W-s

T = Temperature (in °K ≈ 300°K)

P_{avs} = the power available from the source, resonator, in watts

f_0 = oscillation or carrier frequency

f_m = offset frequency

or

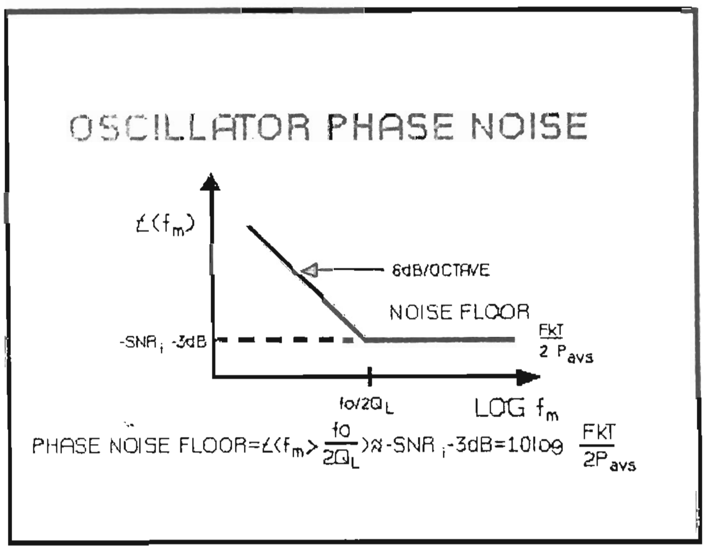
$$\mathscr{L}(f_m) = -10 \log \left[\frac{1}{2}\frac{FkT}{P_{avs}}\left[1 + \left(\frac{1}{f_m}\frac{f_0}{2Q_L}\right)^2\right]\right]$$

or

$$\mathscr{L}(f_m) = -3 \text{ dB} - SNR_i + 10 \log \left[1 + \left(\frac{1}{f_m}\frac{f_0}{2Q_L}\right)^2\right]$$

If we express P_{avs} in dBm, and knowing that thermal noise in a 1 Hz bandwidth = −174 dBm, then

$\mathscr{L}(f_m) = -P_{avs}$ (dBm) + NF (dB) − 177 dBc/Hz + peaking term (dB).



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This slide presents the previous results in graphical form. For large offsets

$$\left(f_m \gg \frac{f_0}{2Q_L} \right),$$

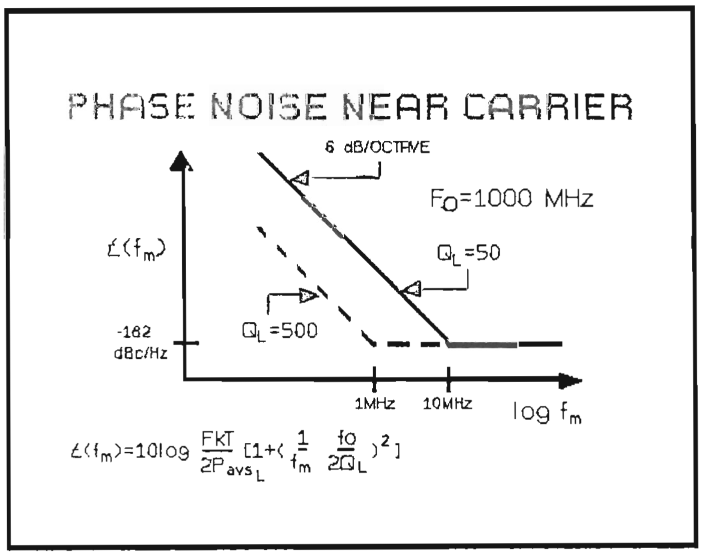
the phase noise floor =

$$\frac{FkT}{2P_{avs}} \approx -SNR_i - 3 \text{ dB}.$$

Inside the offset frequency

$$\frac{f_0}{2Q_L},$$

which is the bandwidth of the open loop circuit of our oscillator model, noise rises 6 dB/octave.



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Let's look at an example. If we assume a power level of +10 dBm and NF = 5 dB then the phase noise floor

$$= -177 \text{ dBc/Hz} + \text{NF (dB)} - P_{avs} \text{ (dB)}$$

$$= -177 + 5 - 10 = -182 \text{ dBc/Hz}$$

$$f_m \gg \frac{f_0}{2Q_L}$$

For phase noise close to the carrier, the equation for shows

$$L(f_m) \approx \frac{1}{2} \frac{FkT}{P_{avs}} \left(\frac{1}{f_m} \frac{f_0}{2Q_L} \right)^2$$

Thus

$$\mathcal{L}(f_m) \propto \frac{F}{P_{avs}} \frac{f_0^2}{Q_L^2} \propto \frac{f_0^2}{SNR_i\, Q_L^2}$$

To minimize phase noise we must maximize signal to noise ratio and loaded Q. Also notice that low phase noise is easier to achieve at low carrier frequencies.

In the example f_0 = 1000 MHz, P_{avs} = +10 dBm, NF = 5 dB, and Q_L = 50.

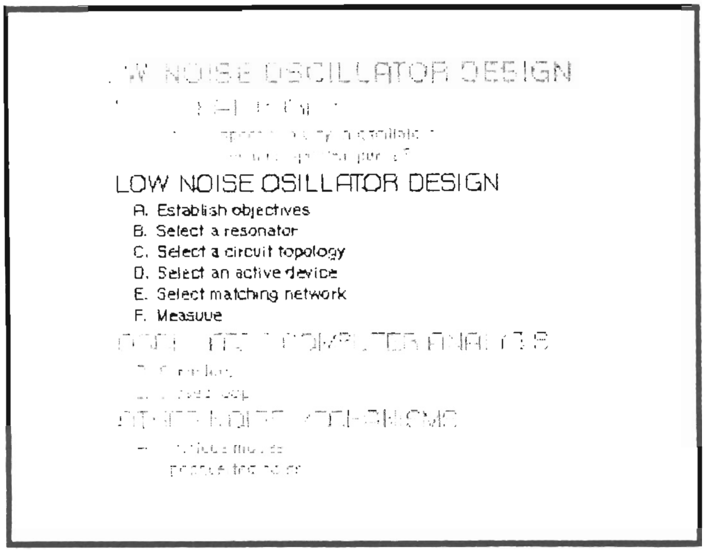
What is $\mathcal{L}$(100 kHz)?

$\approx$ phase noise floor + peaking

$$\approx -182 \text{ dBc/Hz} + 20 \log \left(\frac{f_0}{2f_m\, Q_L} \right)$$

$\approx$ −142 dBc.

If Q_L = 500, this improves 20 dB to −162 dBc/Hz.



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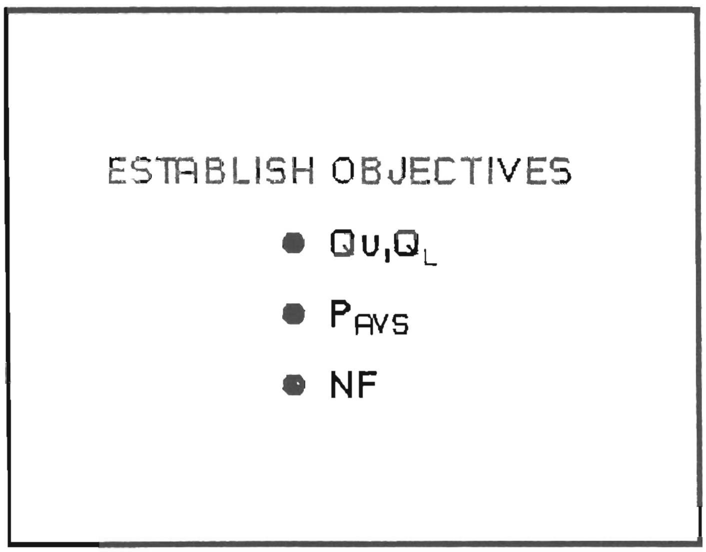

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Starting with a specific close-in phase noise requirement, $L(f_m)_{required}$, then Q_L can be determined from

$$L\left(f_m \ll \frac{f_0}{2Q_L}\right)_{required} \approx \frac{FkT}{2P_{AVS}}\left(\frac{f_0}{2f_m\ Q_L}\right)^2$$

$$\therefore Q_L \text{ required} > \sqrt{\frac{FkT}{2P_{AVS}}}\ \frac{F_0}{2f_m\sqrt{L(f_m)_{required}}}$$

Having Q_L, a resonator may be selected using a rule of thumb that $Q_U \geq 2$ to 5 times Q_L.

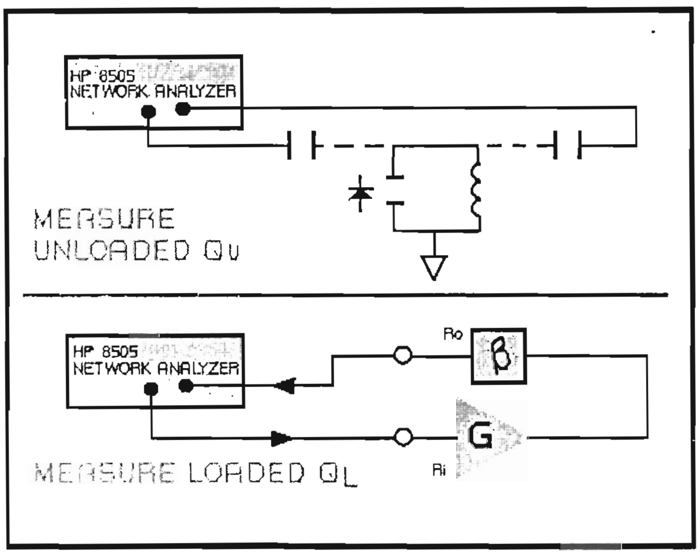
The power level, P_{avs}, is typically set by limitations in the resonator (higher power means greater AM-FM, and potential spurious responses, aging, etc.) or by NF or power handling limitations in the active device

$$\text{Phase Noise Floor} = L\left(f_m \gg \frac{f_0}{2Q_L}\right) \approx \frac{FkT}{P_{AVS}}$$

$$\therefore P_{avs} \gtrapprox \frac{FkT}{L\left(f_m \gg \frac{f_0}{2Q_L}\right)}$$

This relationship may also give a NF requirement

$$F \lessapprox \frac{P_{AVS}\ L\left(f_m \gg \frac{f_0}{2Q_L}\right)}{kT}$$



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Once a potential resonator has been selected, it makes sense to verify some of its parameters, notably its unloaded Q (Q_U), 1/f noise, and AM-to-FM conversion. The unloaded Q of a resonator can be measured on a network analyzer by coupling very lightly to the resonator and measuring either the 3 dB bandwidth, phase slope, or the group delay. For this purpose:

$$Q_U = \frac{f_0}{BW_{3\ dB}}$$

$$Q_U = \frac{f_0}{2}\,\frac{\delta\phi}{\delta f} = \pi f_0\,\tau_{GD}$$

where

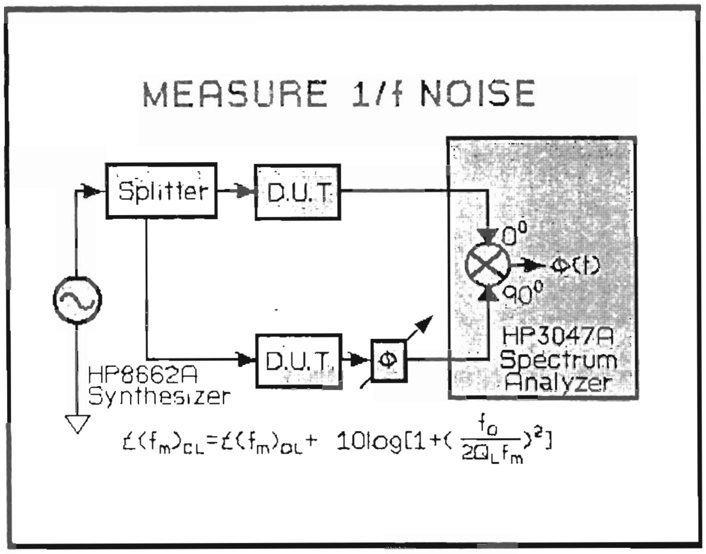
$$\tau_{GD} = S_{21} \text{ group delay in seconds}$$

$$\tau = Q_U/f_0\pi$$

One helpful way to measure Q_U versus frequency is to set the network analyzer in the log frequency mode and draw a 6 dB/octave slope line falling off with frequency. If the group delay rises above the slope line, then Q_U is rising with frequency.

Verifying Q_L in the actual oscillator circuit is also possible, provided that the characteristic impedance Z_0 of the network analyzer is near the operating circuit impedance, or that transformers are used to match the impedances of the test system and the circuit over the frequency range, and power levels are near operating conditions.

Ref. 12, 25, 26, 27



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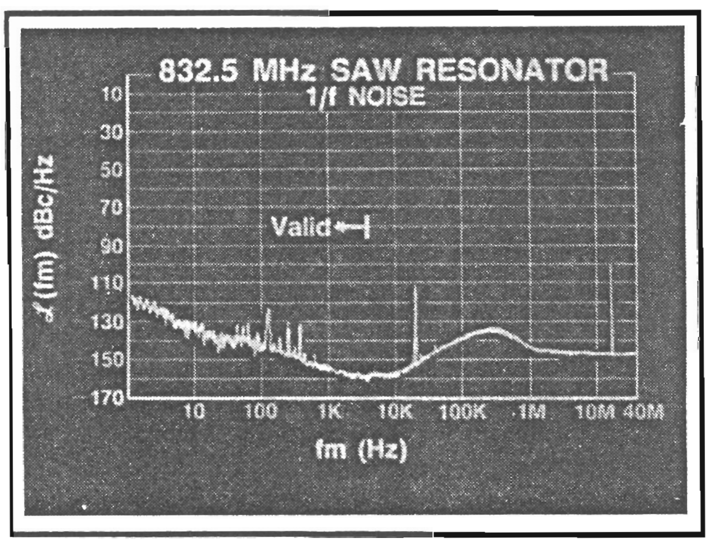
While the calculations presented so far are useful, we have ignored the issue of spectral purity degradation due to 1/f noise.

A spectrum analyzer (or model HP 3047 phase-noise measurement system) can be used to measure the 1/f noise of a resonator or amplifier in a VCO circuit. If two identical resonators and/or amplifiers are used, the group delay difference between the components must be held small in order to prevent the decorrelation of source noise. If identical and independent noise is assumed from each of the two resonators or amplifiers (uncorrelated), then 3 dB must be subtracted from the 1/f noise measured or three-point interpolation (see ref. 3) may be applied. Note, 1/f noise is a multiplicative process, hence the measurement is not typically level dependent; it would still be good to make the measurement at typical operating power, and verify at several different power levels.

Ref. 3.



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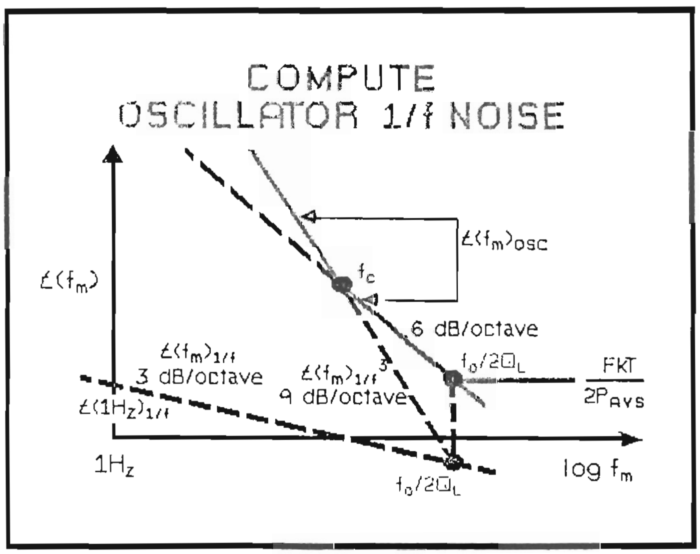
This is an example of 1/f noise measurement using the technique in the previous slide. These data are indicative of typical 1/f noise seen in SAW resonators; we've seen 5 to 10 dB better and 20 to 30 dB worse.

D. Halford has suggested a "rule of thumb" phase noise intercept of −115 dBc/Hz at a 1 Hz offset. See ref. 16.



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It is possible to predict the phase-noise performance of the oscillator circuit (closed loop) by adding the white phase-noise component, $FkT/2P_{avs}$, to the 1/f component, $L(f_m)_{1/f}$, and then modifying both of these by the oscillator closed-loop gain peaking, $[1 + (f_0/f_m 2Q_L)^2]$. The total oscillator phase noise is

$$\mathcal{L}(f_m) = 10 \log \left[1 + \left(\frac{f_0}{f_m 2Q_L}\right)^2\right] \cdot \left[\frac{FkT}{2P_{avs}} + L(f_m)_{1/f}\right].$$

The components of 1/f and white phase noise of the amplifier-resonator combination are artificially separated. However, if we measure the total phase noise, $\mathcal{L}(f_m)_{OL}$, of the series amplifier-resonator open loop (with the correct terminating impedance and power levels), it's possible to predict the oscillator phase noise directly:

$$\mathcal{L}(f_m) = 10 \log \left[1 + \left(\frac{f_0}{2f_m Q_L}\right)^2\right] L(f_m)_{OL}$$

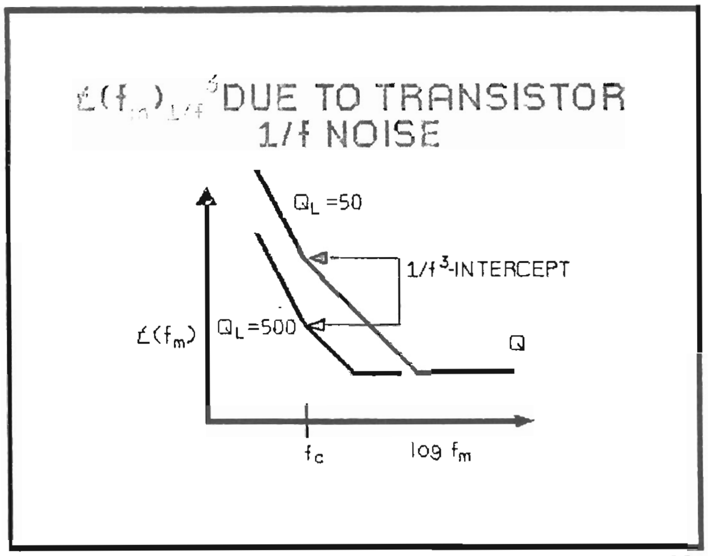
$$\mathcal{L}(f_m) = \mathcal{L}(f_m)_{OL} + 10 \log \left[1 + \left(\frac{f_0}{2f_m Q_L}\right)^2\right]$$

This phase-noise prediction can be shown more easily with a graphical approach. First, plot the phase noise due to white noise components. Then, draw $\mathcal{L}(f_m)_{1/f}$ on the same graph. Next, draw a −9 dB octave line that intersects $\mathcal{L}(f_m)_{1/f}$ at $f_m = f_0/2Q_L$. The intersection of this line with the locus of

$$\mathcal{L}(f_m) = 10 \log \frac{FkT}{2P_{avs}} \left[1 + \left(\frac{1}{f_m} \frac{f_0}{2Q_L}\right)^2\right]$$

is f_c, the $1/f^3$ noise-corner frequency. The 9 dB/octave line then serves as the predicted value of $\mathcal{L}(f_m < f_c)$.

Ref. 41



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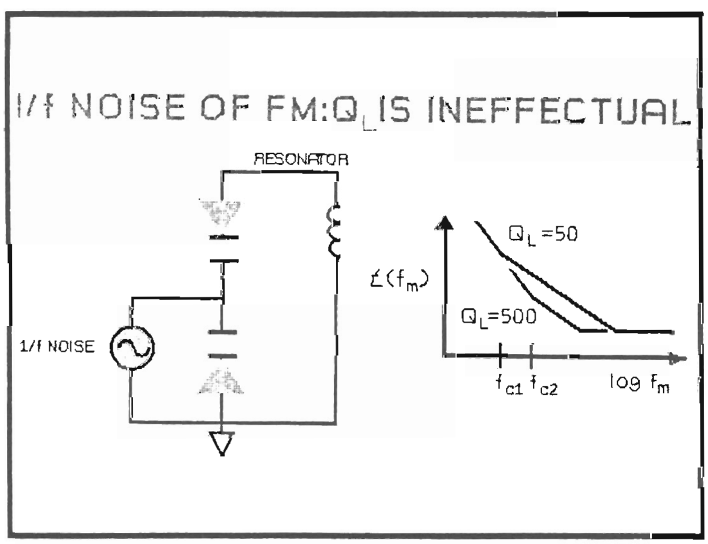
If 1/f phase noise modulation is in the resonator or active device, then an increase in Q_L will improve the phase-noise performance in the 1/f region. This occurs because the loop peaking effect operates on 1/f noise as well as white noise as can be seen from the previous equations.



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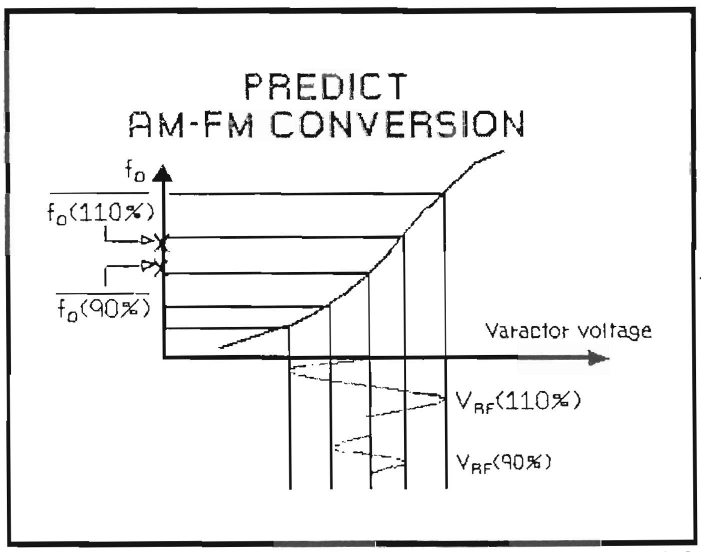
However, if the 1/f noise mechanism is frequency modulating the resonator center frequency, then no improvement of Q_L will lower phase noise in the 1/f region. If a noise source is phase modulating the oscillator, then changing the phase slope of the resonator—or changing the Q—will affect the depth of modulation.

In many VCOs, the spectral purity is dominated by AM-to-FM conversion mechanisms, rather than the SNR_i and Q_L. One method to predict the AM-to-FM conversion in a diode-tuned VCO is by studying the frequency-versus-tuning-voltage characteristic. A change in the rf voltage amplitude on the resonator can affect the average bias on the varactor. A 10-percent change in resonator rf voltage corresponds to 10-percent AM on the carrier. To measure the effects of changing carrier level, one can increase or decrease the rf voltage on the resonator by changing the bias current in the active device. Measure the carrier frequency at 90-percent resonator voltage and compare this with the average carrier frequency at 110-percent resonator voltage. The peak-to-peak frequency shift due to 10-percent AM can then be estimated:

$$\overline{f_0(90\%)} \cong \frac{f_0(+\text{peak}) - f_0(-\text{peak})}{2}$$



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$$\Delta f_{pk}(10\%\ \text{AM}) = \frac{\overline{f_0(110\%)} - \overline{f_0(90\%)}}{2}$$

$$K_v(\text{AM/FM}) = \frac{\Delta f_{pk}(10\%\ \text{AM})}{10\%}\ \text{Hz pk/\% AM}$$

This equation provides a solution in frequency modulation/percent AM. The percent AM, A, should be no less than the collector bias current shot noise fluctuations divided by collector bias × 100 percent:

$$A\%\ \text{AM} = \frac{\sqrt{2}\sqrt{2qI_c}}{I_c} \times 100\% = 200\sqrt{q/I_c}\,\%$$

where

$\sqrt{2qI_c}$ = the full collector shot noise,

and

q= the charge of an electron = 1.6×10^{-19} Coulomb

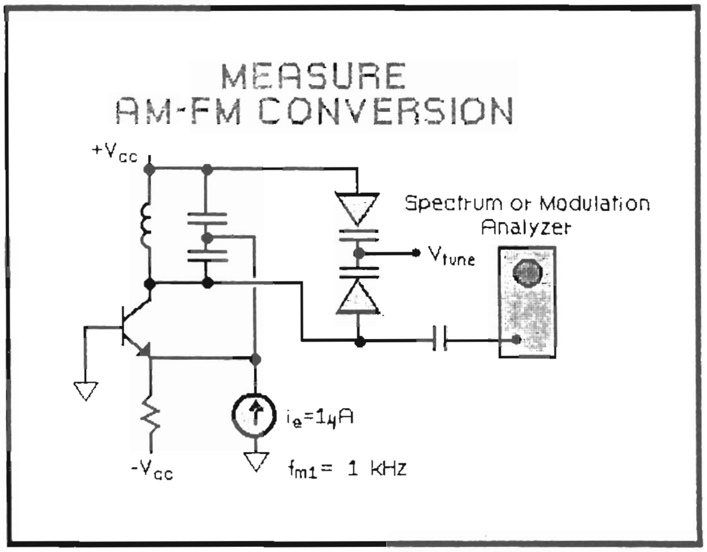
Now we can predict the closed loop phase noise contribution due to AM-FM:

$$\mathcal{L}(f_m)_{AM} = 20\log\left[\frac{A\ K_v(\text{AM/FM})}{2\ f_m}\right]$$

or total phase noise:

$$\mathcal{L}(f_m) = 10\log\left\{\left[1 + \left(\frac{f_0}{2f_mQ_L}\right)^2\right] \cdot \left[\frac{FkT}{2P_{avs}} + L(f_m)_{1/f}\right] + \left[\frac{A\ K_v(\text{AM/FM})}{2f_m}\right]^2\right\}$$

Ref. 28, 29



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There are several methods for measuring the AM-to-FM conversion coefficient ($K_{vAM\text{-}FM}$). In one case, the resonator must be set up at the appropriate power level with the correct amount of coupling/loading, and connected to a network analyzer. By shifting the power level ±10 percent, and monitoring the center-frequency shift.

$$K_v(AM/FM) = \frac{\Delta f_{pk}(10\%\ AM)}{10\%}\ \text{Hz pk/\%AM}$$

Another method of measuring AM-to-FM conversion is to adjust the transistor bias current 10%, monitor Δf, and use

$$K_v(AM/FM) = \frac{\Delta f_{pk}(10\%\ AM)}{10\%}\ \text{Hz pk/\%AM}$$

Typically, a transistor is collector-current cutoff-limiting, so a 10-percent increase in the collector bias current will cause a 10-percent increase in the resonator voltage. The latter approach may cause a change in the active device's phase angle, but this is acceptable, since it's desirable to characterize the sum of all effects contributing to AM-to-FM conversion.

The AM-to-FM conversion can also be tested dynamically. This technique involves injecting a small, low frequency (f_i) current into the transistor's emitter. Adjust f_i until the sidebands around the carrier roll off 6 dB for each octave increase in f_i (which indicates FM):

$$P_{\%AM} = \left(\frac{\text{injected current peak}}{\text{emitter bias current}}\right) \times 100\%$$

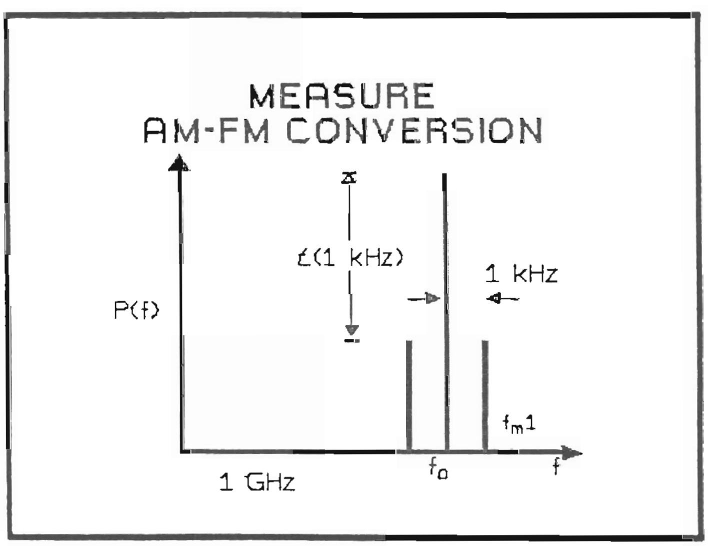
$\mathcal{L}(f_i)$ = measured SSB-to-carrier ratio of the injected FM sidebands. From the narrowband FM approximation we have:

$$\mathcal{L}(f_i) = 20 \log \left(\frac{\Delta f_{ipk}}{2f_i}\right)$$

$$\Delta f_{ipk} = (2f_i)\ 10^{\mathcal{L}(f_i)/20}$$

Δf_{ipk} = peak frequency deviation indicated by these sidebands

$$\therefore K_v(AM/FM) = \frac{\Delta f_{ipk}}{P_{\%AM}}\ \text{in Hz peak/\%AM}$$



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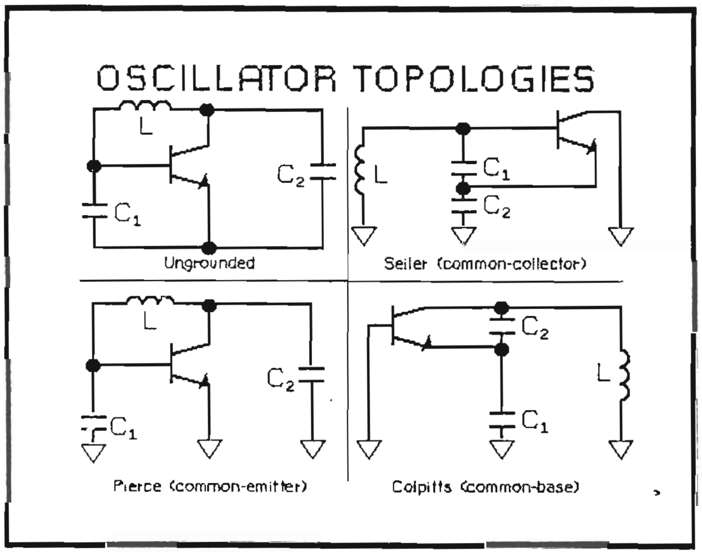
One can also measure this FM modulation directly using an 8901 modulation analyzer.



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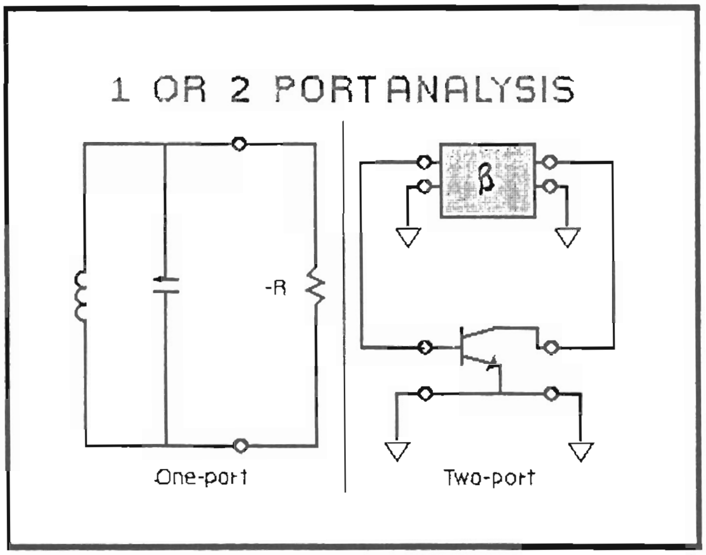
"Many oscillators can be reduced to a Colpitts configuration." The basic layout is an oscillator circuit without a ground terminal. By grounding this circuit at any of its nodes, it can be transformed into any of the other configurations. The preferred topology is one that makes it possible to visualize such things as loop gain, loop phase angle, and stopband stability.

The common-emitter (Pierce) topology is ideal for good out-of-band stability. It yields open-circuit stability at frequencies above about $f_T/3$, and can be kept stable at lower frequencies. Alternatively, the common-base (Colpitts) topology typically has negative real-part impedance at its emitter from about $f_T/5$ to f_{max}, depending on the base-to-ground parasitic inductance. The common-collector configuration, with capacitive loading on its emitter, typically possesses a negative real-part impedance at its base over a significant range of frequencies. Instability is a potential problem whenever there is a negative real part of the impedance at frequencies other than the desired oscillation frequency. The result can be spurious oscillation, squegging, parametric effects, and sharply nonlinear tuning characteristics, especially when a harmonic of the desired frequency crosses through a region of negative resistance.

Refs. 9, 10, 4, 5, 6



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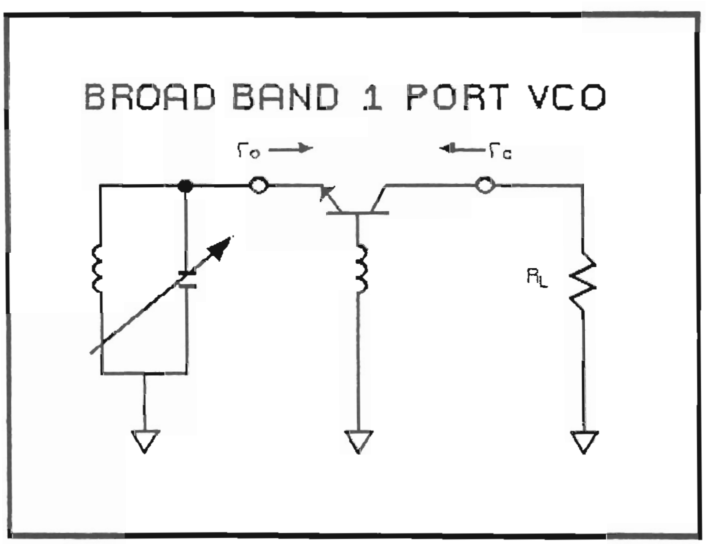
Oscillator topologies can be designed as one-port or two-port configurations. One-ports (negative-resistance oscillators) have good track records in the gigahertz region. The two-port topology permits analysis and ease of visualization using feedback theory; loop gain and phase slope may be more easily derived (and measured) to predict loaded Q_L and spectral purity.



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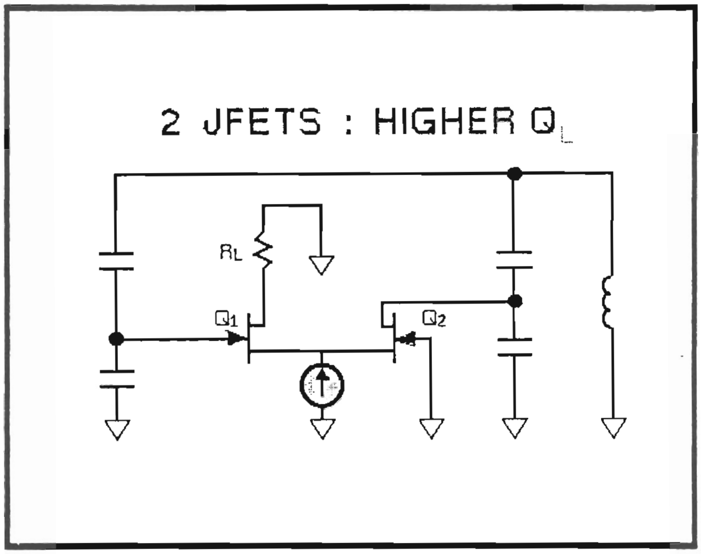
This one-port configuration is widely used in the gigahertz region with YIG-tuned oscillators and VCOs; multi-octave tuning range is a key advantage.

Despite its good points, however, this configuration has its drawbacks. It does not allow easy definition of loaded Q for the purpose of predicting phase noise, nor does it permit simple modeling of loop gain. This topology is also susceptible to spurious modes, since the conditions for the emitter relection coefficient, $\Gamma_e > 1$, leads to potential instability over a broad range of frequencies. The phase noise for this kind of oscillator can be accurately predicted by a computer method we will discuss shortly.

Refs. 17, 18, 19, 20, 21, 22, 23, 31, 32, 33



982

The choices are many: bipolar junction transistors, JFETs, SiMOSFETs, GaAs FETs, Gunn/IMPATT diodes, or miniature packaged amplifiers. In all cases, the selection criteria should include low noise figure at the maximum operating junction temperature; low noise figure at higher power in order to get the highest signal-to-noise ratio (SNR) possible; and low noise figure at the source impedance presented to the device. Certain warnings are also in order. Beware of large ripple occurring in small-signal S_{21} gain in the presence of a large signal at f_0; this indicates nonlinear compression. This is not a parameter that the device manufacturer will specify and must be measured on a network analyzer. Also, be wary of limitations in resonators, such as spurious content in YIG resonators (≥ +10 dBm) and aging in SAW and bulk crystal resonators (≤50 μW is typical for most frequency standards).

Of the devices listed above, the bipolar junction transistor is a natural for low-noise design due to its well-characterized and repeatable parameters. The characteristics of the other devices tend to be not quite so predictable. FETs, for example, exhibit significant variations in pinchoff voltage and performance with temperature. A good rule of thumb is to select f_T at least two to three times the operating frequency, and to remember that the noise figure degrades (i_n increases) as f_0 exceeds f_β.

Ref. 7

A JFET is a good choice for achieving low-noise oscillator performance at $f_0 \leq$ UHF. This performance is most likely due to the high input real-part impedance, which allows tight coupliing and little loading of the resonator (high Q_L). Concurrently, a good noise figure can be achieved with a high source impedance because the JFET input noise current (i_n) is so low:

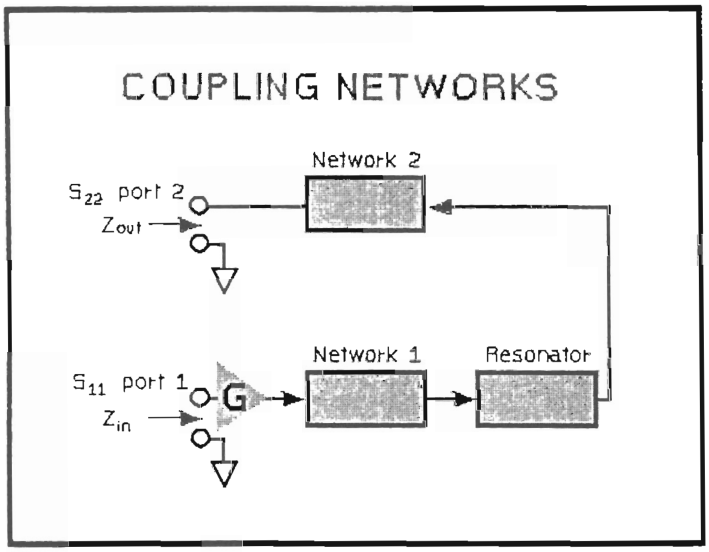
$$r_s \text{ (optimum NF)} = e_n/i_n$$

The end result is high SNR_i or very good phase-noise characteristics.

It has been mentioned that phase noise may be dominated by SNR_i and Q_L (ignoring 1/f noise for the moment). Good SNR_i and Q_L depends on the noise figure of the active device at the operating source impedance, on P_{avs}, and on the Q_L/Q_U degradation due to active device and output loading. The JFET possesses operating characteristics that enable it to achieve high Q_L/Q_U and SNR_i simultaneously.

In a two-port oscillator, there are three contributors to Q_L degradation: the input resistance of the amplifier, the output resistance of the amplifier, and the load resistance. One way to improve Q_L/Q_U and SNR_i is to use two devices in an oscillator circuit. This two-device circuit lightly loads the resonator due to the high input and output real parts of the JFET impedance. The load is isolated from the resonator by Q_1, thus removing the third contributor to Q_L degradation.

At low frequencies especially, take advantage of excess device gain to keep impedances large by using feedback. This will help to not load the resonator Q.



983

The purpose of an oscillator's coupling networks are: to match the input/output impedance of the active device to that of the resonator for optimum P_{avs}, Q_L, and NF; to provide enough phase shift to achieve 0-deg. phase in the angle of the loop-gain transfer function at f_0, where hopefully the loop gain is greater than 1.0 and near the maximum phase slope; and to select the desired operating frequency mode in a multimode oscillator. Some common forms of coupling networks are presented in refs. 12, 9, 10, and 37.

Three coupling network design objectives can also be stated mathematically as

$$|S_{21}|_{\text{loop gain}} > 0 \text{ dB}$$

and

$$\angle S_{21}|_{\text{loop gain}} = 0°$$

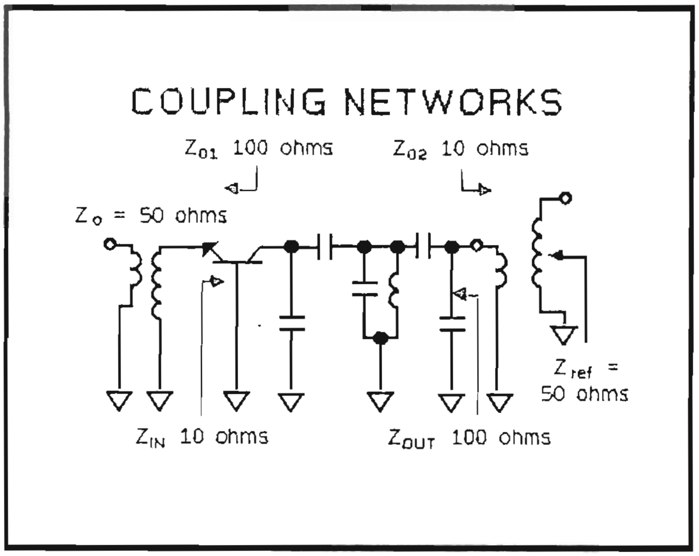
$$\text{for } f = f_0 \text{ only;}$$

and

$$|\Gamma_{\text{node}}| < 1.0$$

$$\text{for } f \neq f_0 \text{ and all nodes.}$$

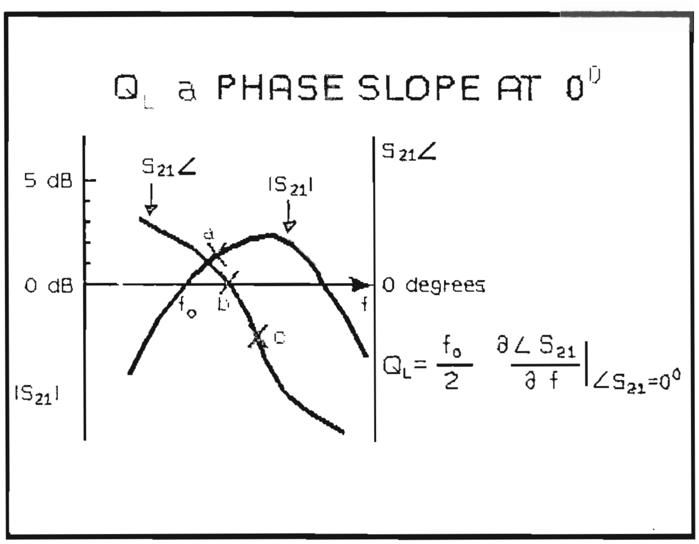
Ref. 12, 9, 10, 37, 38



984

The S-parameter treatment is convenient for use with network analysis. The measured S-parameter data can be used in computer modeling and analysis, and for comparing measured and predicted performance.

A coupling network can be tested with the setup shown. In this slide, Z_{in} is the reference impedance for S_{22} at the output port, and Z_{out} is the reference impedance for S_{11} at the input port. The technique is exactly correct if $Z_{01} = Z_{out}|_{Port\ 2}$ and $Z_{02} = Z_{in}|_{Port\ 1}$. Other conditions are that Z_{in} be measured with Port 2 terminated in Z_{in}, while Z_{out} be determined with Port 1 terminated in Z_{out}. These conditions are not that easy to achieve; still, if the loop is broken where the impedances are reasonably well characterized (real), and ideal (computer-simulated) transformers are employed to get different input and output reference impedances, a model develops which provides fairly accurate loop gain/phase data.



985

From the previous model we get

$$|\text{loop gain}| \approx |S_{21}|$$

$$\angle \text{loop gain} \approx \angle S_{21}$$

The results may appear as those shown, where

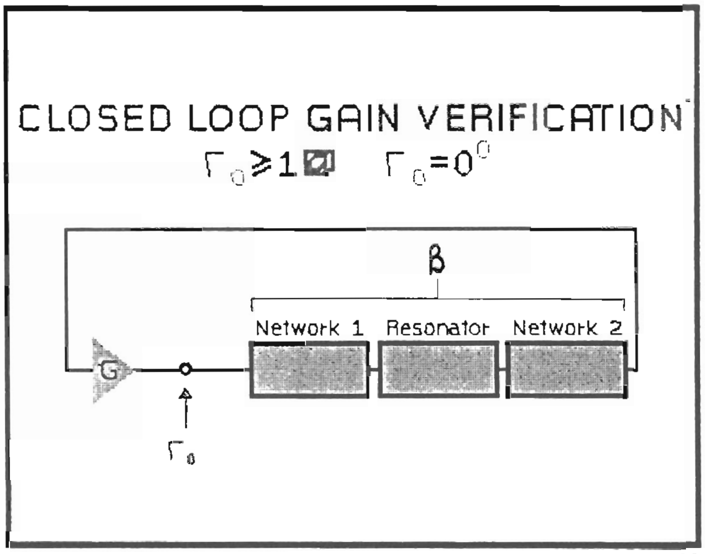
$$Q_L = (f_0/2)(\partial\phi/\partial f)|_{\phi=0°}$$

and

$$\phi(f) = \angle S_{21}(f)$$

It's apparent from this example that oscillation, point b, ($\angle S_{21} = 0°$) will not occur at the maximum phase slope, point c. Consequently, Q_L and the phase noise will be unnecessarily degraded. There is, however, sufficient loop gain (2 dB at point a) for oscillation.

Adjustments to a coupling network make it possible to achieve maximum Q_L, that is, $\angle S_{21} = 0°$ at the maximum phase slope. Coupling to the resonator can also be reduced (so that $|S_{21}| \approx +3$ dB at $\angle S_{21} = 0°$) in order to increase Q_L. Recall that this action may have deleterious effects on P_{AVS} (the power available from the source in dBm) and noise figure as functions of the source impedance.



986

Another test (calculated or measured) for the effectiveness of a coupling network is to close the loop and analyze the reflection coefficient (Γ) at any node. This slide illustrates this concept. Looking at the output of the oscillator, the necessary condition for oscillation is

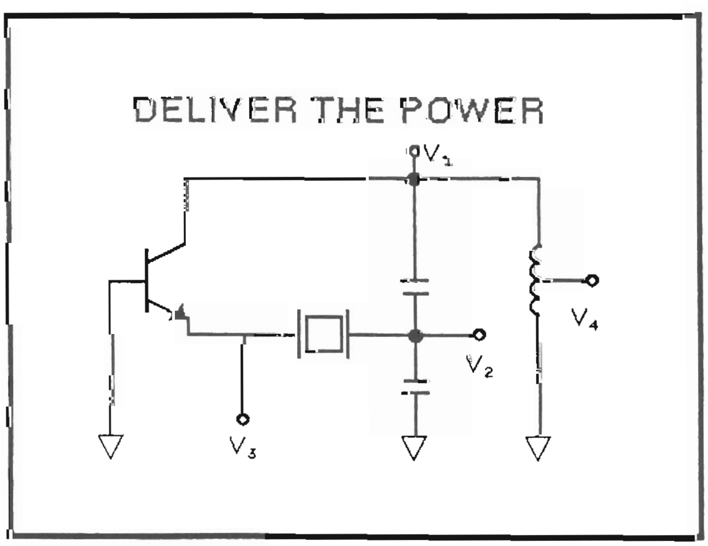
$$\Gamma_0 > 1 \,|_{\angle \Gamma_0 = 0°}$$

where

$$f_0 \approx f \,|_{\angle \Gamma_0 = 0°}$$

Ref. 20

Once there is enough loop gain and the correct phase angle in a design, it's time to consider how to deliver power to the load. Power is typically taken from the resonator, but for the sake of flexibility, it should also be possible to tap signal power at any point in the oscillator loop. Tapping power at V_1, V_2, or V_4 may provide a high signal level to drive limiters and maintain a good noise floor. Node V_4 may have reduced harmonics content due to the lowpass filtering effect of the inductor. Taking power from V_3 may provide a lower noise floor due to some filtering created by the stopband rejection of the resonator crystal.

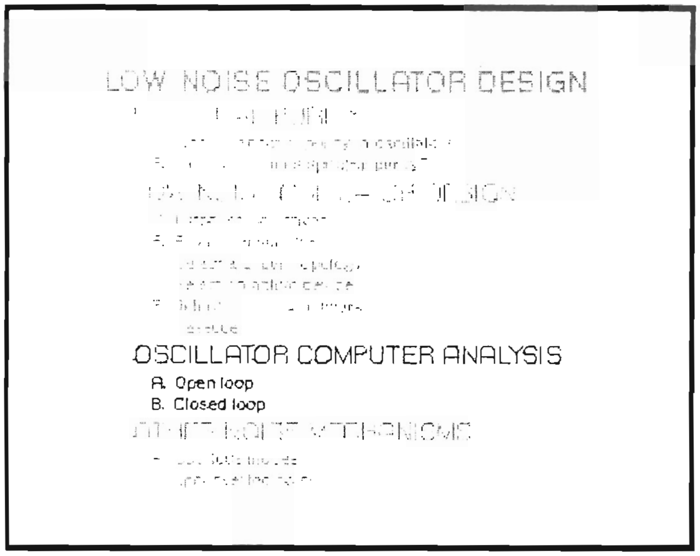


987

Another power-tapping technique is to reflect a load resistance, r_L, to a desired output node, such that r_L is much greater than the real-part impedance seen looking back into that node at f_0. This can be done with matching networks or transformers. As a consequence, the loop gain (and Q_L) is reduced (less than 3 to 6 dB), and the output power may not be significantly reduced.

Tapping power from the collector can provide out-of-band stability by reducing the real-part impedance as seen by the collector (common-emitter and common-base topologies). This creates heightened rejection of undesired modes.

There are many techniques for matching to a load. One method relies on series-to-parallel transformations, ref. 37.



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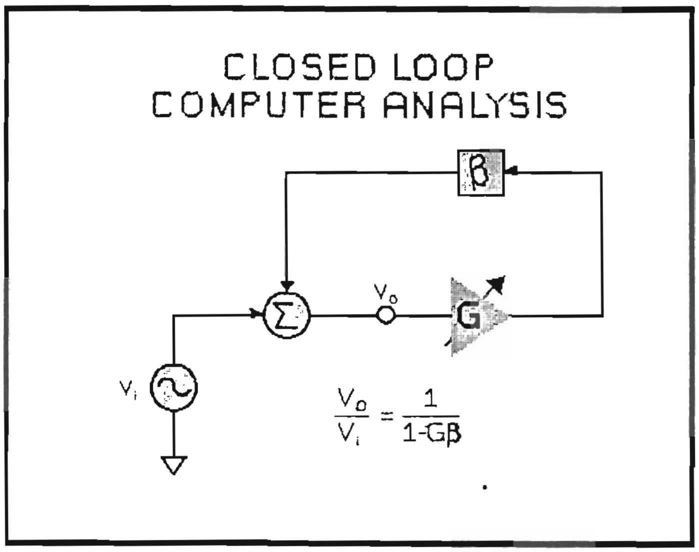
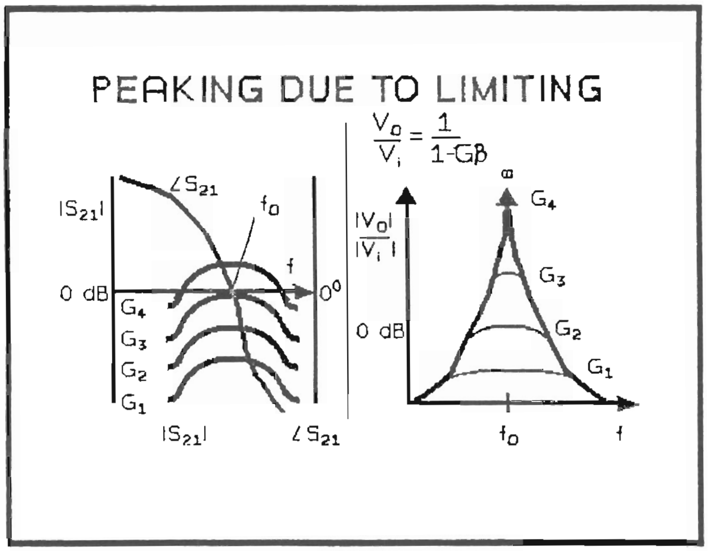


988

A computer helps to evaluate oscillator circuit output-noise spectral density (phase noise) and signal power in a closed loop format by using linear, frequency-domain analysis techniques. This approach is really just an extension of classical feedback control theory. An oscillator is a feedback amplifier whose poles of closed-loop gain transfer function have moved into the right half-plane. Feedback amplifiers may be analyzed for noise and transfer function for any degree of peaking as long as the poles remain in the left half-plane (resulting in no oscillation). If an oscillator is analyzed with its loop gain adjusted for poles very near the $j\omega$ axis, the output noise spectral density will be essentially the same as if the poles were exactly on the $j\omega$ axis (resulting in oscillation).

Since this method accurately predicts and uses actual operating power levels, then P_{AVS} does not have to be known apriori. The computer easily handles the changing NF as a function of rapidly changing source impedance near resonance since we are using actual noise generators in our modeling (not an assumed NF).



989

These figures demonstrate in more detail the process which allows linear-frequency-domain computer analysis to predict closed loop phase noise. If we focus on f_0 where $\angle S_{21} = 0°$ and just modify $|S_{21}|$, then the closed loop gain becomes:

$$\frac{V_0}{V_i}(f_0) = \frac{1}{1 - G\beta} = \frac{1}{1 - |S_{21}|}$$

and we see as $|S_{21}| \rightarrow 1.0$ then

$$\frac{V_0}{V_i}(f_0) \rightarrow \infty$$

goes to infinity. Note the shape of the closed loop gain peaking at the +3 dB corner,

$$f_m = \frac{f_0}{2Q_L},$$

changes very little whether the peak is 40 dB or 90 dB or ∞. Very little is gained by focusing on very close in phase noise

$$\left(fm \ll \frac{f_0}{2Q_L}\right)$$

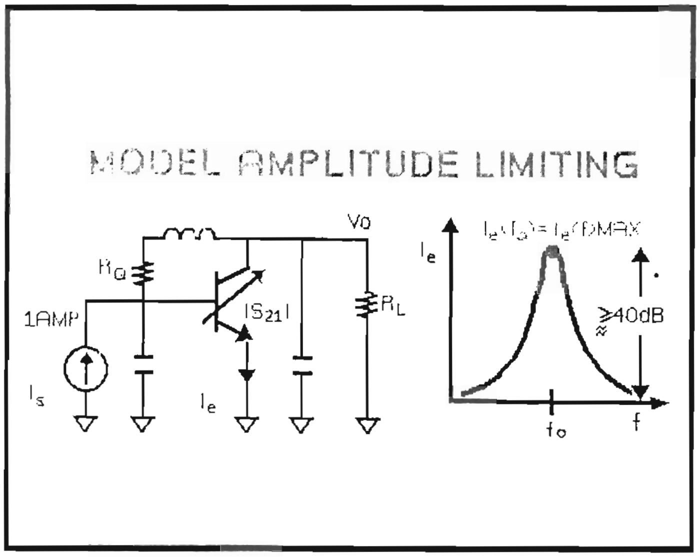
because the shape will remain a constant 6 dB/octave unless we are investigating the effects of crossing high Q spurious modes.

Ref. 11

A basic modeling procedure for predicting parameters like phase noise, P_{OUT}, or node voltages and branch currents follows eight steps:

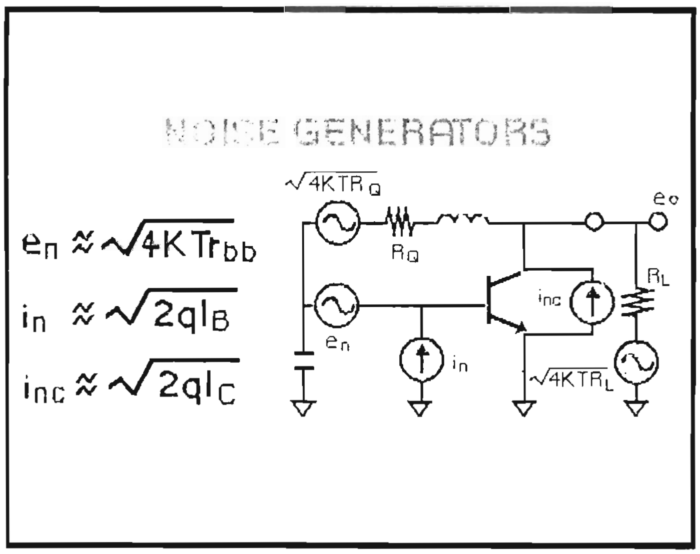
- choose a limiting mechanism (e.g., collector current) for modeling the oscillator. Typically, adjust $|S_{21}|$ of the active device to model the collector current cutoff limiting.
- inject a current source into any node.
- adjust the gain $|S_{21}|$ to model the limiting mechanism so that the closed-loop gain peaking is greater than 40 dB.
- monitor the emitter current at resonance during the computer analysis and scale the computer-analyzed value to the limiting current actually found or predicted in the circuit.
- scale all node voltages and branch currents by this factor. This provides output voltage, resonator voltage, and any other branch current or node voltages during oscillation.
- remove the current source and add all appropriate noise voltage and current sources.
- plot the spectral density of the output noise.
- review the ratio of the output voltage to the output noise (in a 1 Hz bandwidth) gives the predicted SNR_i, hence the phase noise:

$$\mathcal{L}(f_m) = -SNR_0(f_m) - 3\text{ dB (for PM only)}$$



990

This transistor oscillator is biased for collector-current cutoff-limiting (no saturation) operation. Experience teaches us that when the transistor goes into compression then $|S_{21}|$ decreases and $\angle S_{21}$ remains approximately the same. As a result, the loop gain variation with level can be modeled through $|S_{21}|$ adjustments alone. A more sophisticated model might use full-blown large-signal S-parameters and adjust S_{11}, S_{12}, S_{21}, and S_{22} accordingly (this has not been found necessary to achieve accuracies within 1 to 2 dB).



991

The next step in this example is to adjust $|S_{21}|$ until there is at least 40 dB peaking in output due to I_S. The exact amount of peaking is not critical, so long as

$$20 \log \frac{I_e(f_0)}{I_e(f \gg f_0)} \geq 40 \text{ dB}$$

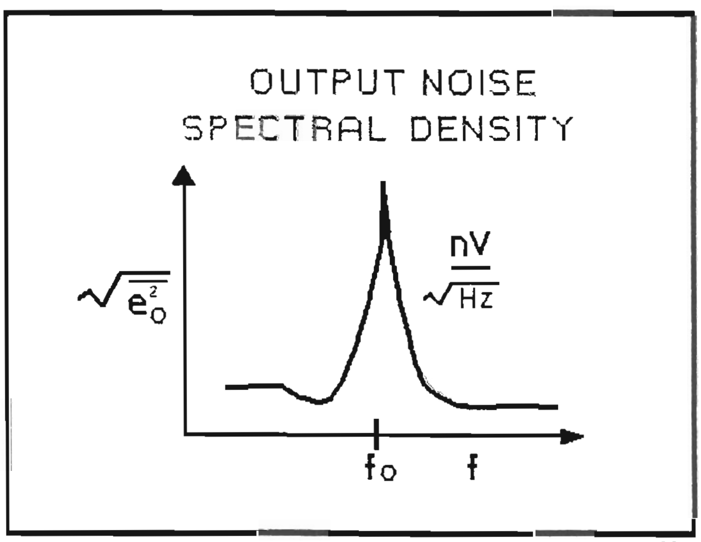
Following this, monitor the emitter current and scale peak value of $I_e(f)$ to the actual emitter bias current, I_E

$$\text{scale} = I_E/[I_e(f)_{max}]$$

With this completed, all node voltages and branch current of interest can be predicted with

$$V_0 = \text{scale} \times V_0(f_0)_{max} = \text{output voltage}$$

$$\left.\begin{aligned} V_N &= \text{scale} \times V_N(f_0) \\ I_B &= \text{scale} \times I_B(f_0) \end{aligned}\right\} \text{ any node or branch}$$

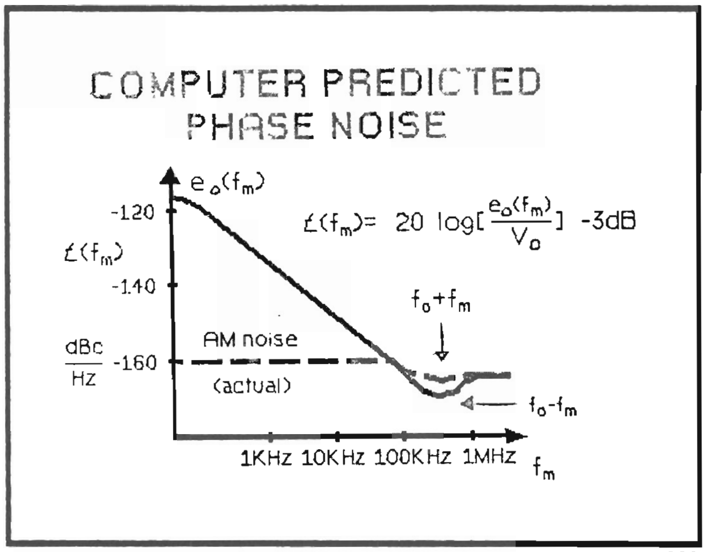


992

The next step is to remove I_S, introduce all noise generators, and plot the spectral density of the output noise voltage. The computer can automatically generate appropriate noise for all lossy elements.

The use of noise current i_{nc} accounts for that component of i_n which increase as f approaches f_T ($f \gg f_\beta$).

Ref. 7, 8



993

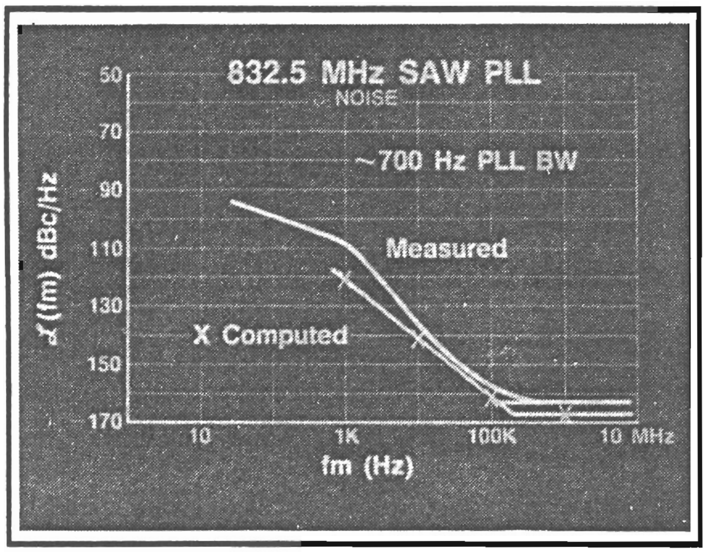
Output phase noise $\mathcal{L}(f_m)$ is simply the ratio of output noise, $e_0(f = f_0 \pm f_m)$, to the output signal voltage, V_0, subtracting 3 dB for the desired phase modulation components only:

$$e_0(f_m) = e_0(f = f_0 \pm f_m)$$

$$\mathcal{L}(f_m) = 20 \log [e_0(f_m)/V_0] - 3 \text{ dB}$$

Unfortunately, this procedure gives no indication of AM noise performance. However, experience has shown that the AM noise is often within 3 to 6 dB of the phase noise floor for

$$f_m \ll \frac{f_0}{2Q_L}$$

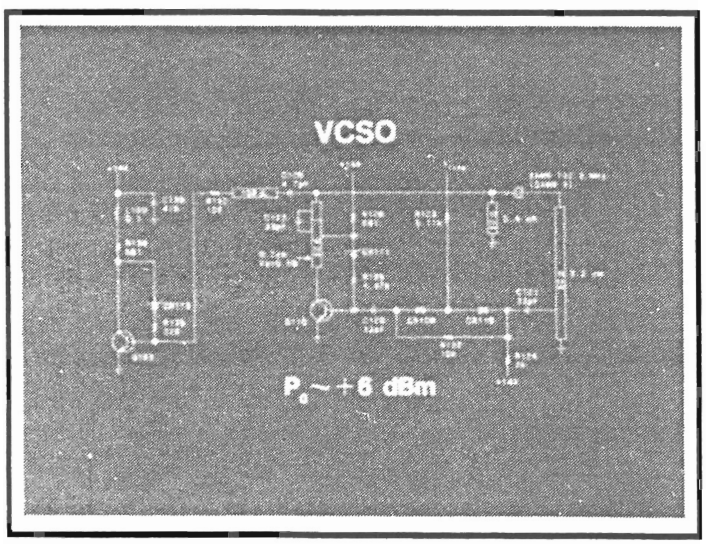


994

These two figures show the computer model and a comparison of predicted results versus measured data. The computer model was even more detailed, including parasitic reactances and e_i and i_n. Small signal S-parameter data were used to model the transistors.

The discrepancy below 10 kHz is due to SAWR 1/f noise and PLL residual noise. The discrepancy above 100 kHz is due to noise contributions of 4 buffer-limiter stages which were not modeled; measurements in phase noise floor with the buffers removed indicate ≈ −165 dBc/Hz (computer suggests −167 dBc/Hz).

The advantages are that we have a precise and controllable experiment with which to better understand the "whys" behind the oscillator performance and predict worse case conditions.



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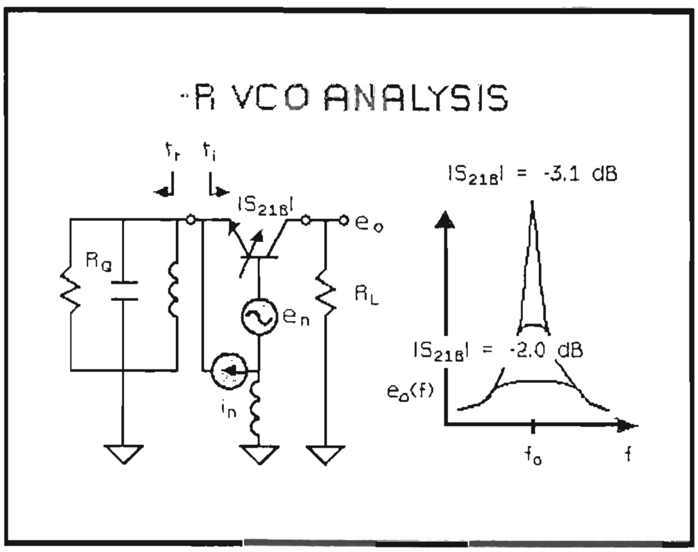


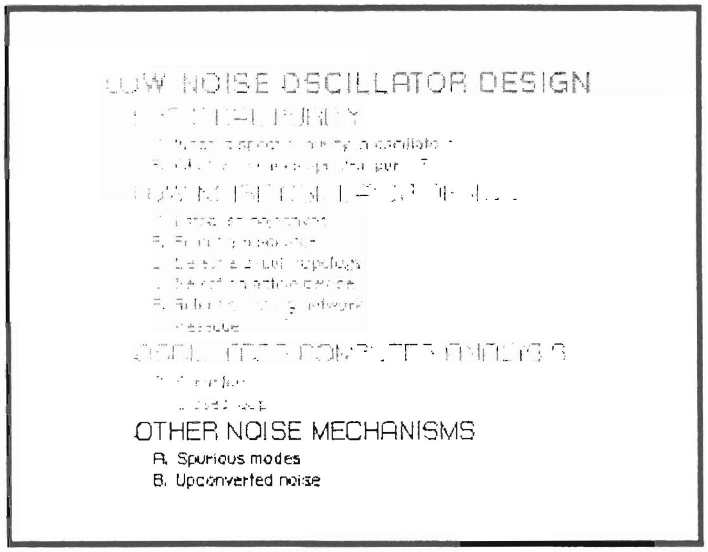
1072

One-port or negative resistance type oscillators may be handled similarly. The procedure involves modeling the oscillator in the manner applied to two-port oscillators, adding in all noise generators, and adjusting $-R$ or $|S_{21b}|$.

The adjustment of $|S_{21b}|$ to achieve better than 40 dB peaking can be automated if an analysis program has optimization capability. The technique requires searching for a peak near resonance and optimizing $|S_{21b}|$ for maximum peaking. It should be kept in mind that f_0 changes slightly with $|S_{21b}|$.

$$S_{21b} = \text{common base } S_{21}$$

Other Noise Mechanisms



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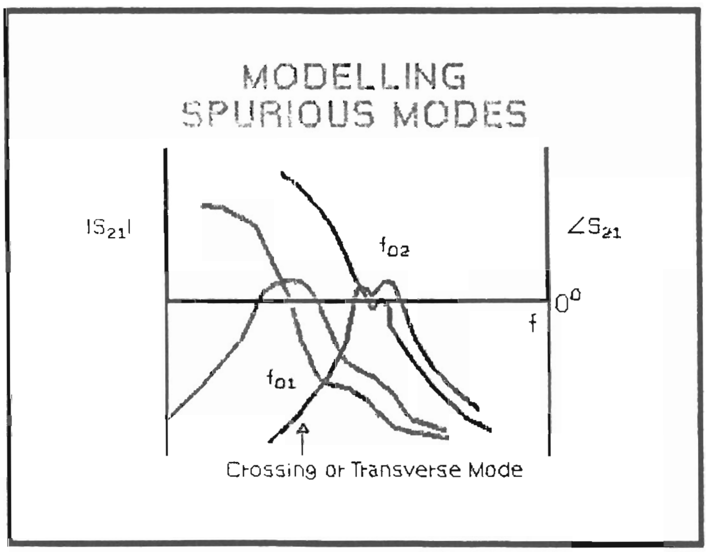
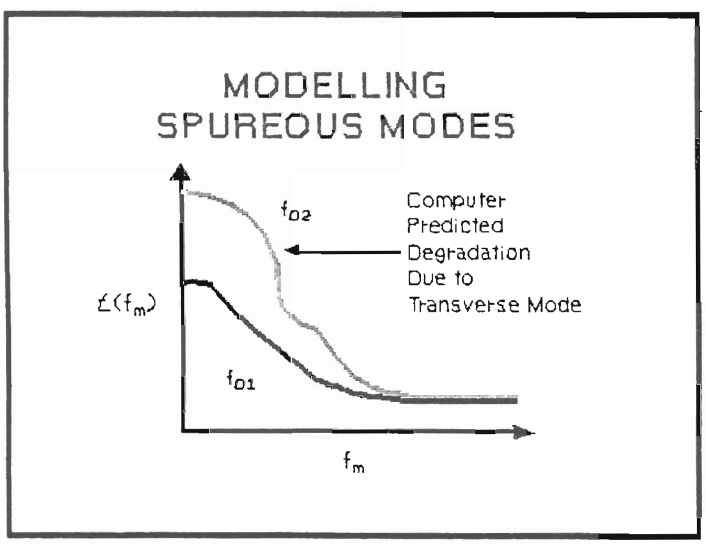


997

Computer analysis also allows modeling spurious modes (such as transverse, crossing, and tracking modes) in a resonator (model or use measured S-parameter data). The technique consists of repeating the phase noise analysis either near (f_{01}) or coincident (f_{02}) with the undesired mode. In this way, it's possible to see the blooming effect on phase noise due to degradation in phase slope for a coincident mode, or phase noise peaking in the noise floor away from the carrier due to an adjacent mode that comes within 3 to 6 dB of the loop gain of the desired mode.

These analyses were computed from a SAWR oscillator using S-parameter data of the SAWR which has a transverse mode approximately 200 ppm above the desired mode.



1074

A more complicated noise degradation mechanism is low-frequency noise upconverted around the carrier. Low frequency noise contributors (1/f base current noise and e_n or i_n) may cause excess emitter current noise in the audio frequency range if no attention is paid to low frequency source impedance and feedback effects. When the active device is near compression, the upconversion gain for i_e up around the carrier can be as low as −3 to −9 dB. We can measure this by injecting a low level current i_e into the emitter:

Audio upconversion gain, G_C, is

$$G_C = 20 \log \frac{i_e(1\text{ GHz} + 1\text{ kHz})}{i_e(1\text{ kHz})}$$

The phase noise in amplifiers due to low frequency noise $i_{ne}(f_m)$ is then

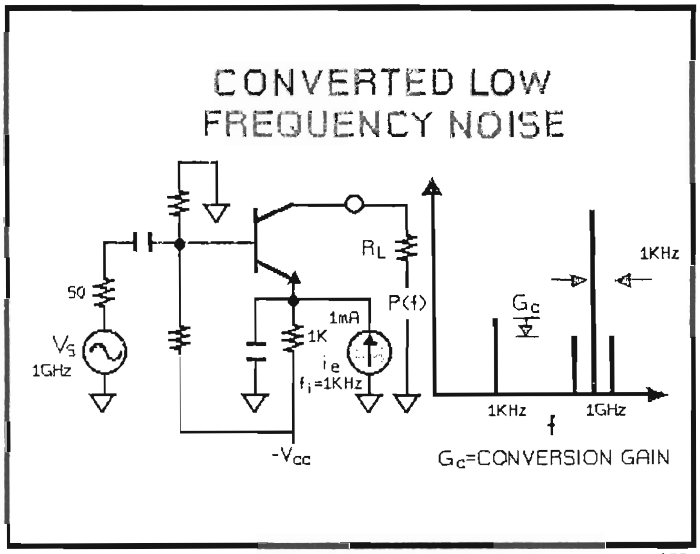
$$\mathcal{L}(f_m) = 20 \log \frac{i_{ne}(f_m)}{I_e(f_0)}$$

$$-3\text{ dB} + \text{upconversion gain}$$

and in the oscillator add the peaking

$$\mathcal{L}(f_m) = 20 \log \frac{i_{ne}(f_m)}{I_e(f_0)}$$

$$-3\text{ dB} + G_C + 10 \log\left[1 + \left(\frac{f_0}{2Q_L f_m}\right)^2\right]$$



1075

To prevent these problems analyze via computer the emitter current noise spectral density of the oscillator and all buffer chains from DC to beyond f_0. Typically, the noise floor due to this mechanism should be 10 dB lower than the specification limit of that contributed by the oscillator.

There appears to be at least two kinds of 1/f noise in transistors: (1) upconverted 1/f component of base current noise; (2) 1/f phase modulation of the RF signal through the transistor. The reason these components seem separate and distinct is that component (1) should be level dependent since it is upconverted by a nonlinear mixing process. Our measurements show that the 1/f PM of active devices remain virtually constant over a significant range of RF power (from limiting to well inside the linear active region [small signal]). Also measuring the 1/f base current noise and the upconversion coefficient we find that the actual residual 1/f phase noise of the amplifier is 20 to 30 dB greater than that predicted by upconversion.

SUMMARY

SPECTRAL PURITY

LOW NOISE OSCILLATOR DESIGN

A. Reonators
B. Circuits
C. Active Devices
D. Matching
E. Measurements

COMPUTER ANALYSIS

OTHER MECHANISMS

1076

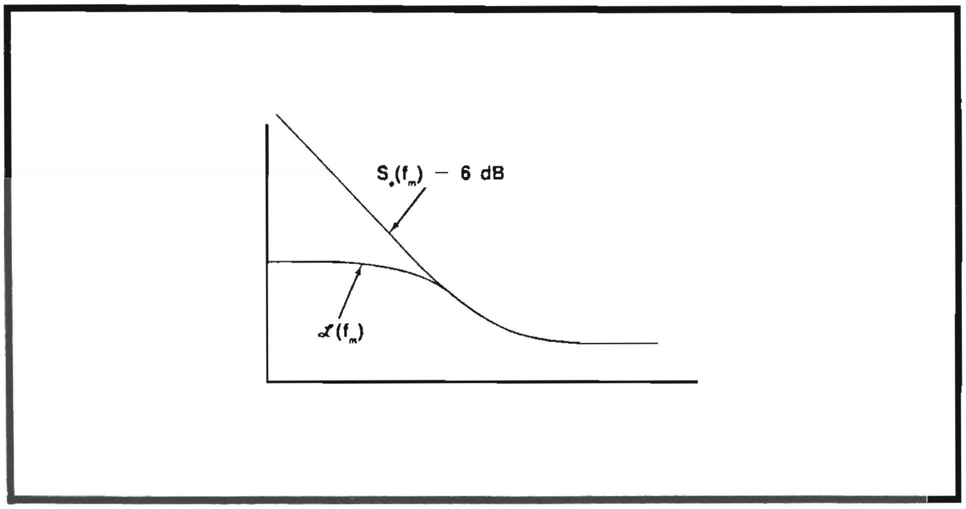
In summary some of the causes of phase noise in oscillators and what to do about them were discussed. The effects of resonator Q, resonator and device 1/f noise, and AM-FM conversion on phase noise were discussed. Oscillator topologies and active devices were looked at. Coupling to resonators and coupling to loads and their effects on noise were examined. Methods of measuring and computer modeling the causes of noise in oscillators were discussed. And lastly, mention was made of other mechanisms that can cause noise.

Appendix 1

$$\mathcal{L}(f_m) = 20 \log \frac{\Delta\phi}{2} \text{ narrowband FM approximation}$$

$$\left.\begin{aligned} L(f_m) &= \frac{S_\phi(f_m)}{2} \\ \mathcal{L}(f_m) &= S_\phi(f_m) - 3\text{ dB} \end{aligned}\right\} \text{ for } \Delta\phi_{total} \ll 1$$

In this article we have assumed $\mathcal{L}(f_m) = S_\phi(f_m) - 3$ dB everywhere for simplicity. Actually as we approach the carrier $\mathcal{L}(f_m)$ flattens out:



Reason: $\mathcal{L}(f_m)$ is the *power* in 1 Hz band centered f_m Hz off the carrier due to PM divided by total signal power. This is what we would see on a spectrum analyzer with a 1 Hz B.W. if reference level (0 dB) was the total signal power.

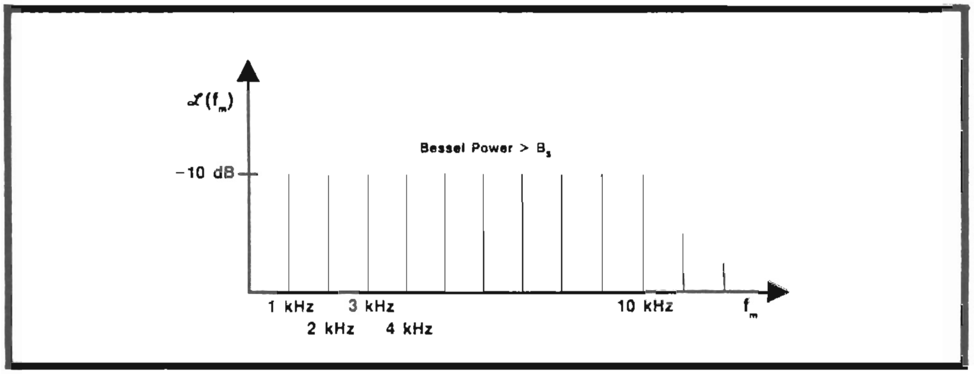
$S_\phi(f_m)$ is the phase spectral density or $20 \log \Delta\phi^2(f_m)$ in the 1 Hz B.W. Here's a simple example which may clarify:

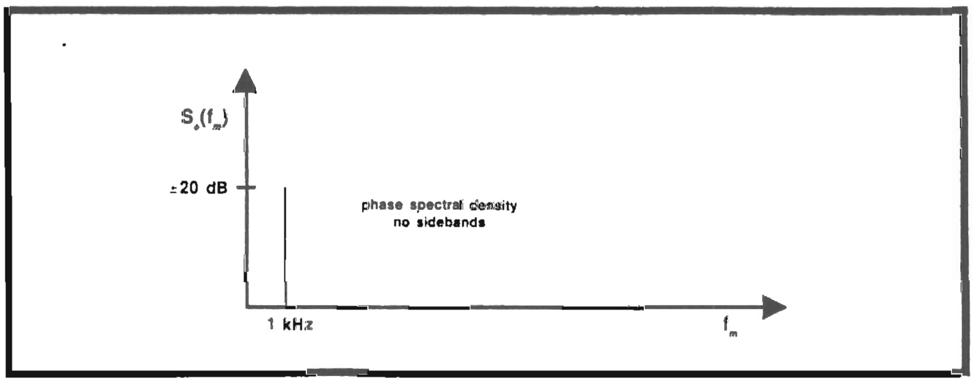
for $\Delta\phi > 1$ radian

Signal: $V(t) = \cos(\omega_C t + 10 \sin 2\pi\, 10^3 t$ or carrier with $\Delta f = 10$ kHz

$fm = 1$ kHz; $\beta = 10 =$ mod index

Appendix 1





$\phi(t) = 10 \sin 2\pi\ 10^3 t$ no harmonics

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LIST OF SYMBOLS

f = offset frequency

FM = frequency modulation

AM = amplitude modulation

S- = scattering parameters

Q_L = loaded Q

SAW = surface acoustic wave

P_{avs} = power available from source (resonator) in Watts

$P_{AVS} = 10 \log P_{avs} + 30$ dBm

Po = output power

CE = common emitter

CC = common collector

CB = common base

Γ = reflection coefficient

ϕ = angle of loop gain

LIST OF SYMBOLS

K_v = VCO gain: Hz/Volt

τ_{GD} = group delay

I_E = emitter dc bias current

I_c = collector dc bias current

I_e = emitter AC signal current rms

f_0 = frequency of oscillation

i_{nc} = collector noise current in $A_{rms}/\sqrt{Hz}$

i_{ne} = emitter noise current in $A_{rms}/\sqrt{Hz}$

$\mathcal{L}(fm)$ = single sideband power in a 1 Hz bandwidth (due to phase noise) referred to signal power in dBc/Hz (see Appendix II)

SNR_i = signal to noise (1 Hz BW) referred to input in dB =

$$10 \log \frac{P_{avs}}{FkT}$$

L(fm): $\mathcal{L}(fm)$ = 10 log L(fm)

NF = 10 log F = noise figure in dB

F = noise factor

e_0 = output noise voltage in volts rms/$\sqrt{Hz}$

G = active device gain

r_S = source resistance

f_T = current gain-bandwidth

K_V (AM/FM) = VCO gain due to AM-FM in Hz/%AM

$\mathcal{L}(f_m)_{OL}$ = open loop phase noise

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